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Contextualisation of Fractions: Teachers' Pedagogical and Mathematical Content Knowledge for Teaching

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The teaching of mathematics for the development of numeracy is emerging to be an important goal of recent reforms. In broad terms, numeracy involves the application of mathematical concepts and procedures in the understanding and solution of a range of problems including real-life problems. Teaching for numeracy, thus calls for skills in translating abstract concepts in mathematics into appropriate real-life contexts and developing an understanding of children's difficulties in this area. In this study, we examined this issue by analysing knowledge of teachers as they attempted to contextualise an abstract fraction problem involving division. Data analyses showed that the participating teachers (n=4) have developed a conceptually weak schema that girds the comprehension and subsequent contextualisation of the given problem. This was evidenced by not only their failure to provide a correct real-life problem representation but also limited knowledge about how they could help students who might have difficulty with similar problems. However, the teachers have developed a robust body of general pedagogical knowledge that was relevant to the teaching of fractions. Taken together the data suggested that their content knowledge for teaching mathematics and pedagogical content knowledge were both weak in this specific area of fractions.

Key words: Fractions; Numeracy; Teachers' content knowledge of mathematics; Teachers' pedagogical content knowledge of mathematics; Contextualisation; Problem representation; Problem modelling

Introduction

It is acknowledged that fractions as a class of numbers are difficult for many students. Fractions are both difficult to teach and learn because these numbers can manifest in different representations but all having a common conceptual basis. As a class of numbers, fractions are different from whole and counting numbers in that students have to recognise parts and wholes. For example, in the number $^1/_2$, 1 and 2 play different roles in comparison to their meaning in whole numbers. The conceptual basis for the correct representation of half involves the recognition that 1 part of a whole that has been divided or shared into two equal parts. In general $^a/_b$ indicates that a whole has been divided into b number of equal parts each representing $1/_b$, and *a* stands for the number of those equal parts. Beyond this basic symbolic understanding, students need to be able to ground this in real-life contexts, a second layer of challenge and difficulty for many students (Mack, 2001).

In this paper, we examine the difficulty in both the teaching and learning of real-life problems that involve fractions. A key factor in the learning outcomes of students is the kind of learning experiences that teachers could provide in order to help them construct and experiment with multiple representations of fractions. However, the richness of such learning experiences is a function of the quality of knowledge that teachers themselves could bring to the learning-teaching context. By elucidating this knowledge, we can engage in a more informed debate about how teachers' knowledge could better be utilised to understand student difficulties with respect to mathematics, in general, and fractions in particular (Ball & Bass, 2000; Hill, Rowan, & Ball, 2005; Chinnappan & Lawson, 2005).

The research reported here examined what a group of teachers knew about fraction problems and their awareness of difficulties experienced by students within an area of fractions. Our aim was to describe teachers' content knowledge and pedagogical content knowledge of fractions. In so doing, we were also interested in finding out the relationship between their content knowledge and pedagogical content knowledge in this area of primary mathematics. The pedagogical content knowledge may be exhibited in many forms including the anticipation of and support for students' learning difficulties.

Background to the Issue

Recent reform movements in the teaching of primary and secondary mathematics have placed a high degree of emphasis on concept development that empowers students becoming more numerate. In order for numeracy to be developed effectively, the teaching of mathematics has to focus on deep understanding of concepts that enables students to situate the concept in a range of meaningful real-life situations (Australian Association of Mathematics Teachers, 2006). In this regard there is consensus that fraction continues to be an area of school mathematics that students tend to struggle with (Stacey et al., 2001). As a class of numbers, fractions has been conceptually challenging for a large number of students. Depending on the context, fractions can be represented as a part of a regional whole, a portion of a discrete set of objects, a measurement point on a number line, or one number divided by another. These multiple avatars that fractions assume render them difficult to grasp (Leinhardt & Smith, 1985).

Failure to grasp the concepts that underpin fractions would have significant impact on students' ability to be numerate. Although there are many areas within mathematics where numeracy needs to be addressed, students' ability at relating fractions to real-life contexts had been identified to be an important area for research (Vale, 2007). This is due to the fact that learners are often called upon to apply their knowledge of fractions in real-life situations. Knowledge of fractions, for example, is applied to a range of tasks including cooking, shopping, building and finances. If students have not developed a sufficient understanding of fractions they could be expected to experience difficulties in solving problems in real-life contexts. This, in turn, not only affects their overall learning but also their job prospects in the future (Marr & Hagston, 2008). In their analysis of the link between numeracy and teaching, Yeh and Nason (2008) suggest that in order for mathematics to be taught more effectively and the concomitant increase in numeracy skills, teachers need to have sound knowledge of the subject matter, and, also use a wider range of tools and techniques in order to provide multiple representations to student.

Theoretical Framework

The dimensions of teacher knowledge that is required for effective mathematics instruction was identified by Hill, Ball and Shilling (2008). In their framework, Hill et al. identified two broad categories: subject-matter knowledge (SMK) and pedagogical content knowledge (PCK). A sub-category of PCK is *knowledge of content and students* that includes teachers' understanding of areas of mathematics that students find difficult to learn. In the present study we aim to examine both the SMK and *knowledge of content and students* (PCK). Sullivan, Clarke and Clarke (2009) highlighted the role of these two categories of knowledge in converting tasks to learning opportunities and suggested this relationship as an important area for future research.

The foregoing review shows that students struggle with fractions and this in turn affects their numeracy levels. Teachers need to develop better knowledge in the area of fractions and they need to provide students with opportunities to apply their knowledge to real-life situations and across other key learning areas – a requirement for numeracy. Accordingly, the aim of this research was to find out what teachers know about fractions (Content Knowledge), their knowledge of students' learning including misconceptions and use of effective learning tasks (Pedagogical Content Knowledge) both of which are instrumental in supporting better engagement with mathematics and development of numeracy (Sullivan et al., 2009).

Research questions

The following research questions guided our analyses of data.

1) How do the teachers contextualise a given Fraction Division Contextualisation Problem (FDCP)?
2) What are the teachers' understandings of children's misconceptions about solving FDCP?
3) What are examples of strategies or approaches that the teachers could use to assist students with misconceptions that emerged in Research Question 2?
4) What are the teacher's views about the cross-curricular implications of teaching children to engage with problems that are similar to FDCP?

Methodology

Participants

A convenience sampling strategy was used to choose the four teachers of primary mathematics who participated in the present study. Two of the teachers, with more than 30 years of classroom experiences each, were considered to be Experienced Teachers (ETs). The other two teachers, with less than 5 years of classroom experiences, were considered to be Less Experienced Teachers (LETs). All participants were drawn from a pool of volunteers who were practitioners at the time of data collection. Table 1 provides a summary of the teachers' background.

Table 1
Participants' Background

Teacher	Gender	Educational background	Grade levels taught	Years of teaching experience
A	Female	Bachelor of Education (Primary)	1-6	32
B	Female	Bachelor of Education (Primary)	1-6	30
C	Female	Bachelor of Education (Primary)	1-6	4
D	Female	Bachelor of Education (Primary)	1-6	5

A and B: Experienced Teachers; C and D: Less Experienced Teachers

Task and Procedure

In order to examine the contexualisation of a fraction problem that involves division (Research Question 1), Item 7 of Content Knowledge for Teaching Mathematics Measures (CKTM) (Ball, Hill, Rowan, & Schilling, 2002) was selected and administered. The CKTM measures are normative and validated with a group of experienced classroom teachers (Hill & Ball, 2004). The instrument provides a reliable measure of teachers' mathematical content knowledge that is used in classrooms. Item 7 (Figure 1) involved the contextualisation of a fraction operation into an appropriate real-life context. This item will be referred to as Fraction Division Contextualization Problem (FDCP) in the remainder of this paper.

In addition, the teachers were also asked the following two questions that were directed at the other three research questions:

Q1: Please suggest difficulties that students might experience in dealing with these types of problems? What are some misconceptions that you expect the students might have in this area of fractions? (Research Question 2)

Q2: Please provide examples of learning tasks that might help students tackle these types of problems? Give a story problem that involves this fraction operation but drawn from another key learning area? (Research Question 3 and 4)

The participants were given FDCP, Q1 and Q2 one day prior to their interview. This strategy was deemed to support the participating teachers to think more deeply about the problem and reflect on the issues that were addressed by Q1 and Q2. The interviews were completed within 20-30 minutes. During the interview, the teachers were told that the researchers were not seeking correct answers but merely trying to understand how they will address problems of this nature in their teaching. Teachers were asked to select the appropriate context problem(s) and provide reasons for their selection. Following this, they were asked to respond to Q1 and Q2. At every stage, teachers were encouraged to seek clarification on any aspect of the activity including terms used in the focus problem and questions.

Which of the following story problems could be used to illustrate

$1\ ^1/_4$ divided by $^1/_2$?

Context a:

You want to split $1\ ^1/_4$ pies evenly between two families. How much should each family get?

Context b:

You have $1.25 and may soon double your money. How much money would you end up with?

Context c:

You are making some homemade taffy and the recipe calls for $1\ ^1/_4$ cups of butter. How many sticks of butter (each stick= $^1/_2$ cup) will you need?

Figure 1. Fraction division contextualization problem (FDCP).

Data and Analysis

Research Question 1: How do teachers contextualise a given Fraction Division Contextualization Problem (FDCP)?

Table 2
Responses to Contextualization

Teacher	Response
A	*Contexts a* & *c*, since they both require fraction knowledge but Context b does not.
B	*Contexts a, b* & *c* relate to the operation as they can all be turned into the same improper fraction.
C	*Contexts a* and *c*, since they are fraction problems, b is not a fraction operation.
D	*Context a*, since there are $1\ ^1/_4$ pies and they have to go between two families, so they get half.

The data collected for this question (Table 2) show that the teachers' SMK in relation to the fraction problem appears to be somewhat limited. The correct answer to this question was *Contexts b* and *c*. All four participants chose *Context a* which suggest that in the contextualisation of FDCP, *splitting* $1\ ^1/_4$ *pies evenly between two families* was interpreted as dividing by $^1/_2$. Although Teachers A and C selected Context *c*, they also decided to (incorrectly) associate *Context a* with the given expression.

Context b relates to operations since to solve the problem teachers would have to multiply \$1.25 by 2 (doubling). The number, 1.25, needed to be recognised as equivalent or translated to a mixed fraction, namely, $1\ ^1/_4$. Teachers were also called upon to understand a structure of multiplication of fractions, that is, multiplying by 2 is the same as dividing by $^1/_2$, which is the inverse relationship between the two operations. Thus, *Context b* can be related to the operation $1\ ^1/_4$ divided by $^1/_2$. The recognition of this relationship is relatively obtuse in comparison to *Context c*. In *Context c,* teachers are required to find how many sticks of butter are needed, given that a stick of butter represents $^1/_2$ cup, and they had to use $1\ ^1/_4$ cups of butter. The problem could be modelled as dividing $1\ ^1/_4$ by $^1/_2$ in order to determine the required number of sticks of butter.

All the participants identified *Context a* as the correct answer. *Context a* involved splitting $1\ ^1/_4$ pies evenly between two families. To obtain the correct representation for *Context a*, $1\ ^1/_4$ had to be divided by 2, so that each family had an equal amount. The participants did not recognise that dividing by 2 is not the same as dividing by $^1/_2$. This confusion may have been the result of the teachers' understanding that they were *halving* the pies between two families. However, to do so $1\ ^1/_4$ had to be divided by 2, not by $^1/_2$!

On analysing the responses, it could be noted that perhaps the participants did not read the question correctly or became confused in its wording, since none of them obtained the correct set of answers (*Contexts b and c*). Teacher B (ET) identified correctly that there was an inverse relationship between dividing by 2 and multiplying by $^1/_2$. This information, it appears, was not used in her decision- making as she went on to say that all three contexts were related FDCP.

Why did Teachers A, C and D *not* recognise that *Context b* could be translated into a fraction problem? We can only assume that they took \$1.25 at face value and failed to see this as a representation of 1 $^1/_4$. This is a concern for teaching alternate models of fractions and its recognition by students in the real world. As a vitally important numeracy skill, one should be able to recognise that fractions can be expressed in different symbolic forms of numbers, especially those involving money since students deal with it almost every day. It is important for students to understand that 25cents is $^1/_4$ of \$1.00 and that 75cents is $^3/_4$ of \$1.00, and thus \$1.25 can be represented as 1 $^1/_4$. If teachers could not recognise this relationship then it would be difficult for them to provide students with the correct representations. The ability to recognise this relationship is essential for teachers as it would assist them to create pedagogical exemplars in various representations to further enrich students' understanding of fractions.

An interesting finding was that when the participants were given the problem to solve, they were able to obtain the correct answers to each of *Contexts a, b and c* but were not able to make a relationship to the given expression. However, they could not recognise which of the given three contexts is related to the operation 1 $^1/_4$ divided by $^1/_2$ suggesting limited conceptual understanding.

Research Question 2: What are the teacher's understandings of children's difficulties and misconceptions about solving FDCP?

We expected the teachers' knowledge of students' difficulties to be influenced by their own understandings of the focus problem as revealed by data in Table 2. Thus a major student difficulty they could have identified would be students' failure in recognising that \$1.25 was the same as 1 $^1/_4$. They could have also highlighted students' relational understanding of the inverse relationship between division and multiplication as problematic.

Although the participants did not have a complete understanding of the focus question, all the four teachers (Table 3) were able to identify a number of difficulties that students could experience when dealing with fraction problems, in general. The difficulties and misconceptions that were highlighted by the participants did not however relate specifically to the focus problem. Instead the difficulties and misconceptions were broadly related to solving all fraction problems. This could be attributed to the teachers' own incomplete representation of the problem in question.

Table 3
Knowledge of Children's Difficulties

Teacher	Response
A	The main difficulty students face is that as the fraction gets smaller the denominator gets larger. It becomes difficult for students to visualise what a fraction would look like and also to visualise parts of a whole.
B	A number of difficulties exist including students knowledge of improper fractions; students knowledge of value of denominators; students knowledge of division; simplifying fractions and value of shares. Students need to have all these knowledge before they start solving a problem.
C	Students tend to have a lot of trouble with denominators. They don't often understand how to convert fractions, nor understand how a mixed numeral is different from a normal fraction.
D	Students become stuck when the question calls for them to change fractions to a common denominator. In the area of fractions students don't like to explore the question to get the answer, rather they ask teachers how to do it, or give up when they can't.

It is important, however, to acknowledge that the participants had high levels of understanding about why fractions can be so difficult for students to grasp, highlighting major difficulties and misconceptions that students encounter. Since the participants were able to highlight these *general* areas of student difficulty, it would seem that teachers would be sensitive to addressing these in their teaching.

Research Question 3: What are examples of strategies or approaches that the teachers could use to assist students with difficulties or having misconceptions with fractions that emerged in Research Question 2?

Data relevant to Research Question 3 is presented in Table 4.

Table 4
Examples to Assist Students

Teacher	Response
A	Using concrete materials and visual aids; allow students to cut up and experiment with whole parts and cut them into smaller parts, creating fractions. Teaching aids like fraction boards help students see how a whole breaks into fractions
B	Teachers have to ensure that students have background knowledge of the difficulties they might encounter with fraction problems. The learning must be scaffolded for complex problems; teachers have to provide opportunities for practice at the knowledge needed to complete fraction problems. Hands-on materials need to be provided so students can physically see the fraction. Relate the problems to real-life situations, e.g. use pizza instead of pie, betting, investments, shopping, etc.
C	Students need the background knowledge to complete the problems, and learning needs to be scaffolded. Students should be provided with visual representations and concrete materials. Teachers can also simplify the problems into smaller steps so students can complete each step to solve the entire problem.
D	Ensuring students understand that visual representations and written fractions are the same. Allow students to go back to basics when they get stuck and draw representations of the fractions they don't understand. Link the fraction problems to real life situations.

All the four teachers outlined a variety of appropriate examples of learning activities to help students tackle fractions problems.

Although their mathematical knowledge may not be sound, their teaching strategies appear to be explicit, detailed and well thought out. This series of examples have high levels of numeracy value and is consistent with Yeh and Nason's (2008) suggestion that in order for mathematics to be taught more effectively and for numeracy skills to develop, teachers need to equip themselves with a wider range of tools and techniques. However, we found out that there was limited evidence of integrating this knowledge with multiple problem representations.

The participants have identified teaching strategies for catering to different learning styles of students. They suggested allowing students to experiment with fractions using concrete materials, thereby providing different visual representations. It is clear that all participants believed in scaffolding and going back to basics if the students do not understand. However, the participants did not provide examples that related specifically to the focus question that we presented. This we believe was a consequence of a partial understanding of the problem.

The teachers acknowledged that concrete materials and visual representations were extremely important when helping students tackle fraction problems. Teacher B commented that there were numerous lesson ideas and resources that could be used in the teaching of fractions but these required considerable investment in time. She went on to explain that teachers were "time poor" and as a result often adopted a chalk-and-talk teaching approach. Consequently, students who needed more elaborate and hands-on visual explanations did not receive sufficient support. This teacher believed that interactive whiteboards could be used to address this issue. Teacher A highlighted that while it was healthy to utilise a number of different concrete materials as teaching aids, one must have access to them including sufficient financial resources.

Overall, teachers have knowledge of a number of appropriate strategies to help students tackle fraction problems in the classroom. They were aware of potential advantages conferred by providing students with multiple teaching aids. However, the level of this support could be limited by time constraints and the availability of resources. Critically, the teachers' incorrect representation of the focus problem that surfaced in the present study could also impose limits on the use of teaching aids.

Research Question 4: What are teacher's views about the cross-curricular implications of teaching children to engage with problems that are similar to FDCP?

Data relevant to the above question were generated by asking the teachers to provide examples of situations where the operation that was embedded in the focus questions could be explored or evidenced in meaningful activities from other key learning areas of the primary curriculum. Responses from the teachers appear in Table 5 with Teacher D unable to provide any suggestion.

Table 5
Cross-Curricular Integration

Teacher	Response
A	Science experiments that measure half distances, e.g. collecting a number of different sizes of balls and determining which one would bounce half as high as a ball that bounces 125cm., also measuring half distances in physical education.
B	Science activities that require students to divide liquids, e.g. $1\frac{1}{4}$ bottles of liquid need to be split evenly into 2 separate containers for two varying experiments. Also cooking lessons that require students to combine different amounts of different ingredients.
C	Physical education activities that allow students to run measured distances, then find half of the distance, quarter of the distance, etc. Also science experiments with liquids, volume and capacity work.
D	No response

Both the experienced teachers (A and B) attempted to generate examples from science that seemed to be partially relevant to the focus question. An example from physical education context by Teacher C could be seen as marginally adequate. All contexts presented by Teachers A, B and C involved the concept of division or sharing denoted by the operation $1\ ^1/_4$ divided by 2. None of these examples however, embodied the expression, $1\ ^1/_4$ divided by $^1/_2$, again due to the faulty modelling of the focus problem by the teachers.

Discussion and Conclusion

The data presented throughout this paper have provided a window into teachers' content and pedagogical content knowledge in one area of fractions. Data relevant to Research Question 1 showed that the participants had a somewhat limited understanding of the focus fraction problem. Although each participant was able to solve problems embedded in *Contexts a, b* and *c* correctly, they were unable to relate these statements to the operation $1\ ^1/_4$ divided by $^1/_2$, which was the goal of the exercise. They were also unable to recognise the inverse relationship between dividing by $^1/_2$ and multiplying by 2, as well as identifying that \$1.25 was equivalent to $1\ ^1/_4$. This shows that they understood each of the three statements but their schema for fractions was not developed sufficiently to uncover the relationship we were looking for.

The above shortcoming in the content knowledge of fractions emerged in the data generated for Research Question 2, which, in turn, shed light on the third and fourth research questions. The participants' failure to correctly answer the focus problem manifested in their failure to identify student misconceptions. However, the range of difficulties identified by the teachers was comprehensive and relevant to fraction problems, in general.

Data in Tables 4 and 5 showed that teachers suggested a number of effective ways to help students tackle fraction problems in the classroom. The teachers felt that students could be helped in the classroom by adopting appropriate scaffolding strategies such as guiding them step-by-step during the solution process. The participants highlighted that visual aids and concrete materials were also an effective way to help students gain a better understanding of fraction problems. However, they acknowledged that not all schools could be expected to have funding and be equipped with a wide range of resources resulting in teachers having to make do with what is available to them. One of the participants also pointed out that time was often a constraint and a barrier for teachers to set up creative lesson ideas.

Marr and Hagston (2008) explain that numeracy is often 'used in an unconscious way, embedded within other tasks, and is often not acknowledged as numeracy' (p. 8). If so, teachers themselves have to be informed about such contexts and be aware of the integration of mathematical concepts and procedures. This knowledge would help them design learning activities for students such that they appreciate the relational aspects of numeracy (Beswick, Swabey, & Andrew, 2008). Teachers who develop these skills in the classroom can be expected to give their students confidence to see where mathematics can be applied in everyday situations. However, teachers with limited mathematical content knowledge and classroom experiences could not be expected to exploit such opportunities (Ma, 1999; Norman, 2005; Walshaw, 2012), as highlighted by responses from the beginning teachers (Table 5) in the present study.

An interesting finding in the present study relates to teachers who were able to solve the problems procedurally and yet failed to relate the expression conceptually to the three context situations presented to them. Thus, we have evidence here, albeit somewhat limited, for the argument that some of teachers' knowledge of mathematical content is primarily algorithmic in nature. Such a knowledge base cannot be expected to support teachers in the provision of conceptually challenging problems for the students. The limited link between teachers' procedural and their conceptual understanding of fractions is ripe for further research (Forrester & Chinnappan, 2011).

We expected the content and pedagogical content knowledge of the experienced teachers to be more extensive and deep in comparison to their less experienced peers but there did not seem to be any significant difference between the experienced and less experienced teachers in their conceptual understanding (content knowledge). However, experience seemed to be significant in teachers' responses when they attempted to activate contexts from other subject areas (Table 5) and scaffolding strategies (Table 4). Notwithstanding, the generalised comments from even the experienced teachers is a cause for concern as the many years in practice seemed to have not contributed to the development of a robust body of content knowledge that is specific to the context of the focus problem here.

Although this paper has provided an insight into teachers' knowledge of a particular fraction problem, how teachers teach fractions to their students, and teachers' understanding of student's misconceptions of fractions, it has limitations. The study was based on a small sample of participants. There is a need for the study to be replicated with a larger sample of participants and varied fraction tasks before one could generalise the findings. A process of convenience sampling was used to select the participants for the present study. This meant that they were not necessarily teachers with high experience levels with the last two years of the Australian primary school system which are Years 5 and 6. Thus, the teachers may have not been as familiar with problems of this nature. It is possible that a study with teachers drawn from a pool of volunteers who had experienced teaching at the upper end of primary mathematics may provide a better account of teachers' content and pedagogical content knowledge than was assessed here.

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